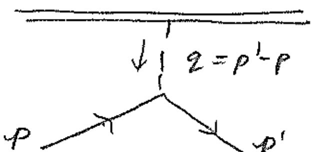
Wavefunction Renormalization in Yukawa Model

The Lagrangian for the Yukawa model with a Dirac spinor field ψ and a real scalar field ϕ is

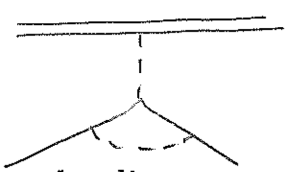
$$\mathcal{L} = i\bar\psi\gamma^\mu\partial_\mu\psi - M\bar\psi\psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\phi^2 - g_0\phi\bar\psi\gamma_5\psi.$$

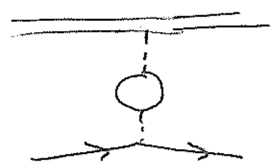
A. Draw the tree-level Feynman diagram for the scattering of the fermion from a very heavy particle by the exchange of the boson. Write down the matrix element $i\mathcal{M}$ for scattering with momentum transfer $q = p' - p$.

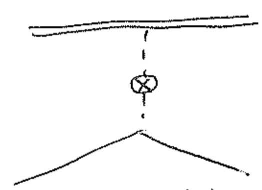
$i\mathcal{M} =$ 

$= G\frac{i}{q^2-m^2}\,\bar u(p')(-ig_0\gamma_5)u(p)$

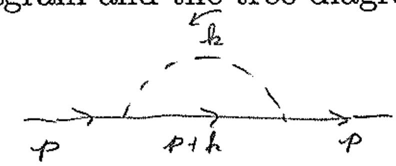
B. Draw the two one-loop diagrams and the tree diagram with a boson self-energy counterterm, labeling the momenta.

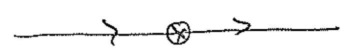






C. Draw the diagrams of order g_0^2 for the fermion self-energy $-i\Sigma(p)$: the one-loop diagram and the tree diagram with a fermion self-energy counterterm.





D. Use Feynman rules to write down the expression for the one-loop diagram.

$$\int\frac{d^4k}{(2\pi)^4}(-ig_0\gamma_5)\frac{i}{\not p+\not k - M + i\epsilon}(-ig_0\gamma_5)\frac{i}{k^2-m^2+i\epsilon}$$

If dimensional regularization is used for ultraviolet divergences, the expansion of the fermion self-energy in powers of $\not p - M$ (in the limit $m \ll M$) is

$$\Sigma(\not p) = \frac{g_0^2}{16\pi^2}\Bigg[\left(\frac{1}{d-4}+\frac{1}{2}-\log\frac{M}{\bar\mu}\right)M + \left(\frac{1}{d-4}+1-\log\frac{M}{\bar\mu}\right)(\not p - M)$$
$$+\big[\text{finite}\big](\not p - M)^2 + \ldots\Bigg] + \Big[\delta M + \delta Z(\not p - M)\Big].$$

E. How must δM and δZ depend on d for $\Sigma(\not p)$ to have a finite limit as $d \to 4$?

$\delta M = \frac{-g_0^2}{16\pi^2}\frac{1}{d-4}M + (\text{finite})$ $\quad \delta Z = \frac{-g_0^2}{16\pi^2}\frac{1}{d-4} + (\text{finite}$

If M is the physical mass, the residue Z_f of the pole in the complete fermion propagator is defined by

$$\frac{i}{\not p - M - \Sigma(\not p)} \longrightarrow \frac{iZ_f}{\not p - M} \quad \text{as } \not p \to M.$$

F. Use the expression for $\Sigma(\not p)$ to determine Z_f.

$$Z_f = \frac{1}{1 - \Sigma'(\not p = M)} = \frac{1}{1 - \frac{g_0^2}{16\pi^2}\left(\frac{1}{d-4} + 1 - \log\frac{M}{\mu}\right) + \delta Z}$$

You can choose to do no fermion wavefunction renormalization: $\delta Z = 0$. The matrix element for scattering of the fermion from a very heavy particle is then

$$\mathcal{M} = G\,\bar u(p')\gamma_5 u(p)\left(\sqrt{Z_f}\right)^2 g_0\left\{1 + g_0^2\left[\Pi(q^2) + \Gamma(q^2)\right] + \ldots\right\}.$$

G. Expand $(\sqrt{Z_f})^2 g_0$ to order g_0^3.

$$\left(\sqrt{Z_f}\right)^2 g_0 = g_0\left\{1 + \frac{g_0^2}{16\pi^2}\left(\frac{1}{d-4} + 1 - \log\frac{M}{\mu}\right)\right\}$$

With on-shell fermion wavefunction renormalization, g_0 is replaced by g_{os}.

H. What is δZ? What is Z_f?

$$\delta Z = \frac{g_0^2}{16\pi^2}\left(\frac{1}{d-4} + 1 - \log\frac{M}{\mu}\right) \qquad Z_f = 1$$

I. Expand $(\sqrt{Z_f})^2 g_{\text{os}}$ to order g_{os}^3. Determine the relation between g_{os} and g_0 to order g^3.

$$\left(\sqrt{Z_f}\right) g_{os} = g_{os}$$

With fermion wavefunction renormalization by minimal subtraction, g_0 is replaced by g_{ms}.

J. What is δZ? What is Z_f?

$$\delta Z = \frac{g_0^2}{16\pi^2}\frac{1}{d-4} \qquad Z_f = \frac{1}{1 - \frac{g_0^2}{16\pi^2}\left[1 - \log\frac{M}{\mu}\right]}$$

K. Expand $(\sqrt{Z_f})^2 g_{\text{ms}}$ to order g_{ms}^3. Determine the relation between g_{ms} and g_0 to order g^3. Determine the relation between g_{ms} and g_{os} to order g^3.

$$\left(\sqrt{Z_f}\right)^2 g_{ms} = g_0\left\{1 + \frac{g_0^2}{16\pi^2}\left[1 - \log\frac{M}{\mu}\right]\right\}$$

$$g_{ms} = g_{os}\left\{1 + \frac{g_{os}^2}{16\pi^2}\left[1 - \log\frac{M}{\mu}\right]\right\}$$