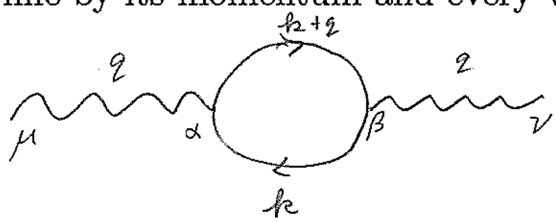
Power Counting for QED

A diagram in QED can be power counted by writing down a factor of $\int d^4k$ for every loop, a factor of $1/k^2$ for every photon propagator, and a factor of $\not k/k^2$ for every electron propagator, and then expressing their product naively as

$$\left(\int d^4k\right)^l (1/k^2)^m (\not k/k^2)^n \;\sim\; \int d^{4l}k\, k^n/k^{2m+2n} \;\sim\; \int^\Lambda k^{4l-2m-n-1}dk.$$

The power counting is $\Lambda^{4l-2m-n}$ if n is even, $\Lambda^{4l-2m-n-1}$ if n is odd, and $\log\Lambda$ if $4l-2m-n=0$. The Feynman rules for QED are $-ig_{\mu\nu}/(q^2+i\epsilon)$ for the photon propagator in Feynman gauge, $i/(\not p - m + i\epsilon)$ for the electron propagator, and $ie\gamma^\mu$ for the vertex.

A. Draw the one-loop diagram for the photon propagator with momentum q, labeling every line by its momentum and every vertex by a Lorentz index.



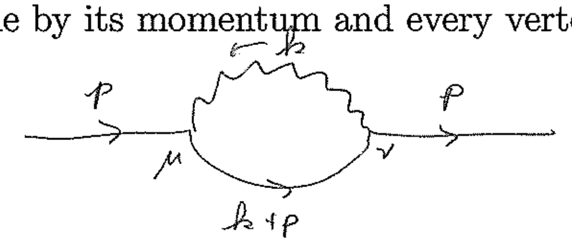
B. Use Feynman rules to write down the expression for the amputated diagram.

$$(-1)\int \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\left(ie\gamma^\alpha \frac{i}{\not k - m + i\epsilon}\, ie\gamma^\beta \frac{i}{\not k + \not q - m + i\epsilon}\right)$$

C. Power-count the diagram and show that it is ultraviolet divergent.

$$\int d^4k \left(\frac{\not k}{k^2}\right)^2 \sim \int d^4k\, \frac{1}{k^2} \Longrightarrow \Lambda^2$$

D. Draw the one-loop diagram for the electron propagator with momentum p, labeling every line by its momentum and every vertex by a Lorentz index.



E. Use Feynman rules to write down the expression for the amputated diagram.

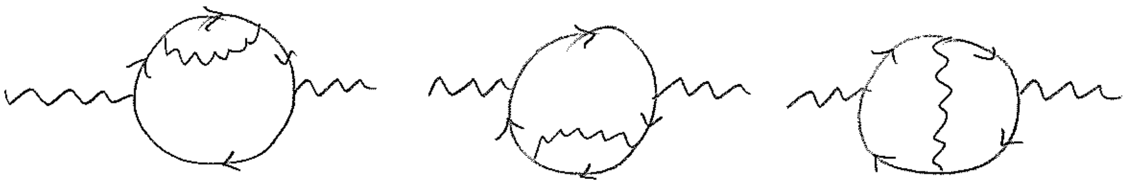
$$\int \frac{d^4k}{(2\pi)^4}\, ie\gamma^\nu \frac{i}{\not k + \not p - m + i\epsilon}\, ie\gamma^\mu\, \frac{-ig_{\mu\nu}}{k^2 + i\epsilon}$$

F. Power-count the diagram and show that it is ultraviolet divergent.

$$\int d^4k\, \frac{1}{\not k}\, \frac{1}{k^2} \sim \int d^4k\, \frac{k}{k^4} \Longrightarrow \log\Lambda$$

For a multiloop diagram, power counting determines the "superficial degree of divergence" from the region where all loop momenta become large, but there can be subdiagrams with a higher degree of divergence.

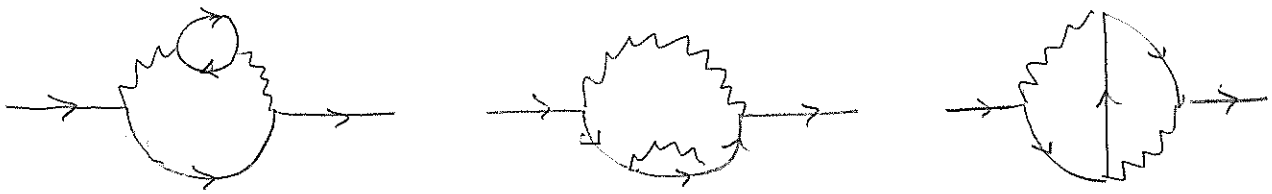
G. Draw the 3 two-loop diagrams for the photon self-energy.



H. Power count any one of the three diagrams.

$$\left(\int d^4k\right)^2 \frac{1}{k^2}\left(\frac{1}{\not{k}}\right)^4 \sim \int d^8k \, \frac{1}{(k^2)^3} \Longrightarrow \Lambda^2$$

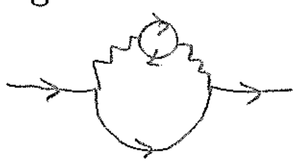
I. Draw the 3 two-loop diagrams for the electron self-energy.



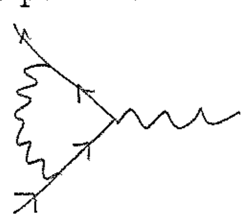
J. Power count any one of the three diagrams.

$$\left(\int d^4k\right)^2 \frac{1}{(k^2)^2}\left(\frac{1}{\not{k}}\right)^3 \sim \int d^8k \, \frac{\not{k}}{(k^2)^4} \Longrightarrow \log\Lambda$$

K. Identify a diagram that has a subdiagram with a more severe ultraviolet divergence than indicated by the power counting.

 is $\log\Lambda$, subdiagram  is Λ^2

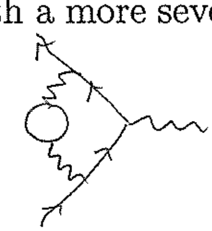
L. Draw the one-loop one-particle-irreducible (1PI) vertex-correction diagram.



M. Power count the diagram and show that it is logarithmically divergent.

$$\int d^4k \, \frac{1}{k^2}\left(\frac{1}{\not{k}}\right)^2 \sim \int d^4k \, \frac{1}{k^4} \Longrightarrow \log\Lambda$$

N. Identify a two-loop 1PI vertex-correction diagram that has a subdiagram with a more severe ultraviolet divergence than indicated by the power counting.

 is $\log\Lambda$, subdiagram  is Λ^2