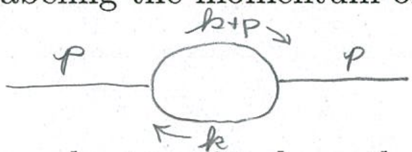
Power Counting for a Scalar Field Theory

The Lagrangian for a scalar field theory with a ϕ^3 interaction is

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu \phi \partial_\mu \phi - \tfrac{1}{2} m^2 \phi^2 - \tfrac{1}{6} g \phi^3.$$

The Feynman rules are $i/(k^2 - m^2 + i\epsilon)$ for a propagator with momentum k and $-ig$ for the vertex.

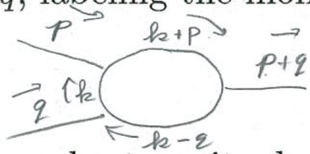
A. Draw the **one-loop propagator-correction diagram** with incoming momentum p, labeling the momentum of every line.



B. Use Feynman rules to write down the expression for the amputated diagram.

$$\frac{1}{2}(-ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(k+p)^2 - m^2 + i\epsilon}$$

C. Draw the **one-loop vertex-correction diagram** with incoming momenta p, q, and $-p-q$, labeling the momentum of every line.



D. Use Feynman rules to write down the expression for the amputated diagram.

$$(-ig)^3 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(k+p)^2 - m^2 + i\epsilon} \frac{i}{(k-q)^2 - m^2 + i\epsilon}$$

A diagram can be power counted by writing down a factor of $\int d^4k$ for every loop and a factor of $1/k^2$ for every propagator, and then expressing their product naively as

$$(\textstyle\int d^4k)^p (1/k^2)^q \sim \int d^{4p}k/k^{2q} \sim \int^\Lambda k^{4p-1} dk/k^{2q} \sim \Lambda^{4p-2q}$$

(or $\log \Lambda$ if $4p - 2q = 0$).

E. Power-count the **one-loop propagator-correction diagram** and show that it is ultraviolet divergent.

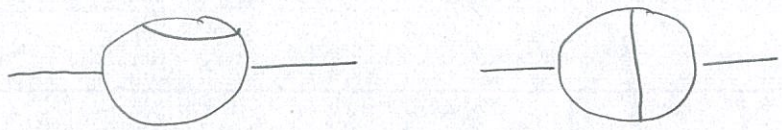
$$\int d^4k \left(\frac{1}{k^2}\right)^2 \sim \int \frac{d^4k}{k^4} \sim \int^\Lambda \frac{k^3 dk}{k^4} \sim \log \Lambda$$

F. Power count the **one-loop vertex-correction diagram** and show that it is convergent.

$$\int d^4k \left(\frac{1}{k^2}\right)^3 \sim \int \frac{d^4k}{k^6} \sim \int^\Lambda \frac{k^3 dk}{k^4} \sim \log \Lambda$$

For a multiloop diagram, power counting determines the "superficial degree of divergence" from the region where all loop momenta become large, but there can be subdiagrams with a higher degree of divergence.

G. Draw the 2 **two-loop** 1PI **propagator-correction diagrams**.

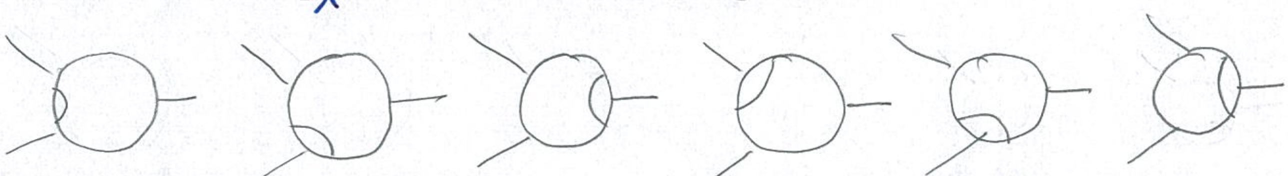


H. Power count any one of the diagrams and show that it is superficially convergent.

$$\left(\int d^4k\right)^2\left(\frac{1}{k^2}\right)^5 \sim \int \frac{d^8k}{k^{10}} \sim \int \frac{k^7dk}{k^{10}} \sim \frac{1}{\Lambda^2}$$

G. Identify the diagram that has a one-loop subdiagram that is logarithmically divergent.

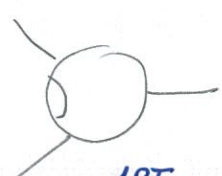
I. Draw the 6 **two-loop** 1PI **vertex-correction diagrams**.



J. Power count any one of the diagrams and show that it is superficially convergent.

$$\left(\int d^4k\right)^2\left(\frac{1}{k^2}\right)^6 \sim \int \frac{d^8k}{k^{12}} \sim \int^\Lambda \frac{k^7dk}{k^{12}} \sim \frac{1}{\Lambda^4}$$

K. Identify a diagram that has a one-loop subdiagram that is logarithmically divergent.



L. Draw the **two-loop** 1PI **vacuum-energy diagram**.

M. Power count the diagram and show that it is ultraviolet divergent.

$$\left(\int d^4k\right)^2\left(\frac{1}{k^2}\right)^3 \sim \int \frac{d^8k}{k^6} \sim \int^\Lambda \frac{k^7dk}{k^6} \sim \Lambda^2$$